sary to determine χ from (4) and substitute its value in (16). If λ_0/D is greater than the right-hand side of (16), side coupling is preferable; if λ_0/D is less, then end coupling will give larger gaps.

Finally, in a practical design it is usually necessary that

$$\frac{\lambda_0'}{2} > D \qquad (19)$$

in order to prevent the generation of TM modes [7] and so minimize loss by radiation. By rearranging (19) and replacing λ_0' by $\lambda_0/\sqrt{\epsilon_r}$, one imposes a further constraint on the permissible range of the λ_0/D ratio for end-coupled filters and for all values of characteristic impedance:

$$\frac{\lambda_0}{D} > 2\sqrt{\epsilon_r}. \qquad (20)$$

To facilitate selection of the filter type, a graph has been prepared with λ_0'/D as a function of $\sqrt{\epsilon_r}\,Z_0$ (Fig. 5), incorporating (14), (16), and (20).

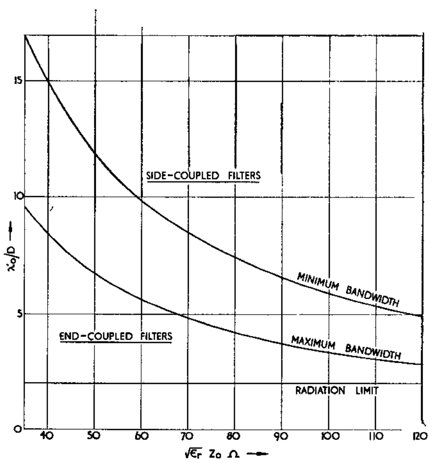

Fig. 5. λ_0'/D as a function of $\sqrt{\epsilon_r}Z_0$ for complete bandwidth range.

ACKNOWLEDGMENT

The permission of the Australian Post Office Research Laboratories to publish this correspondence is acknowledged.

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REFERENCES

[1] S. B. Cohn, "Direct-coupled-resonator filters," *Proc. IRE*, vol. 45, pp. 187–196, February 1957.
[2] E. H. Bradley, "Design and development of strip-line filters," *IRE Trans. on Microwave Theory and Techniques*, vol. MTT-4, pp. 86–93, April 1956.
[3] S. B. Cohn, "Parallel coupled strip line resonator filters," *Onde Elec.* (*France*), pp. 140–146, October 1957.
[4] H. M. Altschuler and A. A. Oliner, "Discontinuities in the Center Conductor of Symmetric Strip Transmission Line," *IRE Trans. on Microwave Theory and Techniques*, vol. MTT-8, pp. 328–339, May 1960.
[5] A. A. Oliner, "Equivalent circuits for discontinuities in balanced strip transmission line," *IRE Trans. on Microwave Theory and Techniques*, vol. MTT-3, pp. 134–143, March 1955.
[6] S. B. Cohn, "Shielded coupled-strip transmission line," *IRE Trans. on Microwave Theory and Techniques*, vol. MTT-3, pp. 29–38, October 1955.
[7] K. S. Packard, "Optimum impedance and dimensions for strip transmission line," *IRE Trans. on Microwave Theory and Techniques*, vol. MTT-5, pp. 244–247, October 1957.

An Interesting Impedance Matching Network

Any load impedance can be transformed to a real impedance by a $\lambda/8$ transformer whose characteristic impedance is equal to the magnitude of the load impedance.

The locus of a normalized load impedance, as a function of the characteristic impedance of the feed line, is a constant (X/R) contour on the Smith chart. Further, a constant (X/R) contour is a circular segment drawn through the points zero and infinity. The point of closest approach of any constant (X/R) contour to the $(Z/Z_0)=1$ point, minimum $|\Gamma|$, takes place along a straight line drawn from $(Z/Z_0)=j$ to $(Z/Z_0)=-j$, or $\lambda/8$ away from the real axis. Therefore, if any load is fed by a line whose characteristic impedance minimizes the magnitude of the reflection coefficient at the load, the normalized impedance $\lambda/8$ away from the load will be pure real.

Consider the matching network in Fig. 1.

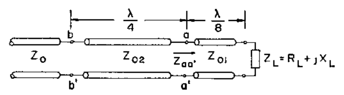

Fig. 1. Load-line impedance match.

The squared magnitude of the reflection coefficient at the load is given by

$$|\Gamma|^2 = \frac{|Z_L|^2 - 2R_LZ_{01} + Z_{01}^2}{|Z_L|^2 + 2R_LZ_{01} + Z_{01}^2} \qquad (1)$$

where

$$R_L = \text{Re}\,\{Z_L\}.$$

We may write

$$|\Gamma|^2 = 1 - \delta \qquad (2)$$

where

$$\delta = \frac{4R_LZ_{01}}{|Z_L|^2 + 2R_LZ_{01} + Z_{01}^2}. \qquad (3)$$

Then

$$\frac{\partial\delta}{\partial Z_{01}} = 0 \qquad (4)$$

implies that

$$(|Z_L|^2 + 2R_LZ_{01} + Z_{01}^2)(4R_L) - (4R_LZ_{01})(2R_L + 2Z_{01}) = 0 \qquad (5)$$

or

$$Z_{01}^2 = |Z_L|^2. \qquad (6)$$

Therefore, $|\Gamma|^2$ is a minimum if $Z_{01}=|Z_L|$. Since $|\Gamma|<1$, $|\Gamma|$ will also be minimized by the same value of Z_{01}.

The driving point impedance of the $\lambda/8$ transformer, $Z_{aa'}$ in Fig. 1 is

$$Z_{aa'} = \frac{R_L}{1 - \dfrac{X_L}{|Z_L|}} \qquad (7)$$

when

$$Z_{01} = |Z_L|.$$

Thus if the Q of the load impedance is high,

$$Z_{aa'} \cong \begin{cases} \dfrac{R_L}{2}, & X_L < 0 \\ 2X_LQ_L, & X_L > 0 \end{cases} \qquad (8)$$

where

$$Q_L = \frac{X_L}{R_L}. \qquad (9)$$

The impedance of the quarter-wave transformer in Fig. 1 may then be found from

$$Z_{01} = \sqrt{Z_{aa'}Z_0} \qquad (10)$$

as is well known.

By application of the same principles, a conjugate match between two impedances Z_G and Z_1 may be accomplished in a $\lambda/2$ length of line as shown in Fig. 2, where

$$Z_{01} = |Z_L| \qquad (11)$$

$$Z_{02} = \left[\frac{R_LR_G|Z_L||Z_G|}{(|Z_L|-X_L)(|Z_G|-X_G)}\right]^{1/2} \qquad (12)$$

$$Z_{03} = |Z_G| \qquad (13)$$

in which

$$Z_L = R_L + jX_L \qquad (14)$$

$$Z_G = R_G + jX_G. \qquad (15)$$

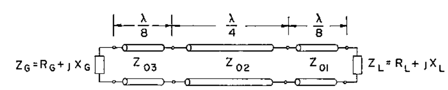

Fig. 2. Source-load impedance match.

These circuits do not constitute minimum length solutions to the impedance matching problem. They do provide a simple answer to the synthesis problem in cases where frequency response is not the primary concern, as is the case in many frequency multiplier design problems. In addition, the line lengths involved are always known in advance. This property has been found to be particularly useful in microwave circuit design when the exact value of the input impedance is not a priori known.

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Manuscript received December 5, 1966.

A Laminar Slow-Wave Coupler and Its Application to Indium Antimonide

The purpose of this correspondence is to describe a technique for coupling energy between a waveguide mode and a semiconductor (or gas discharge) slow wave that has a longitudinal component of microwave electric

Manuscript received January 3, 1967. The research reported here was sponsored by the Air Force Avionics Laboratory, Research and Technology Division, U. S. Air Force Systems Command, Wright-Patterson AFB, Ohio, under Contract AF-33(615)-1608, and by RCA Laboratories, Princeton, N. J.

field. When a bar of semiconductor material which supports such a wave is mounted as a central inductive post in dominant mode waveguide, the energy coupling occurs by way of the microwave electric field existing between the top and bottom waveguide surfaces. If the post is many wavelengths long (i.e., wavelengths of the slow wave within the semiconductor), the contributions from alternate half-wavelength sections of the post will cancel and the net coupling will result from the last full or fractional half-wavelength in the post. The post, viewed as a whole, is thus coupled very poorly to the waveguide.

Consider an inductive post mounted in a series of small-height waveguides stacked atop one another as illustrated in Fig. 1(a). The height of each waveguide is equal to an odd number of half-wavelengths *in the semiconductor*, but can have a width appropriate for dominant mode waveguide. This post would couple to alternate waveguides with 180° phase difference as illustrated in Fig. 1(b). At one end, the individual small-height waveguide sections could all be terminated by a short circuit located so as to present an optimum impedance at the post position. The other (output) end of this structure could have short-circuit terminations on alternate waveguides which would prevent them from coupling to the output waveguide. Thus the unshorted small-height waveguide sections would all couple in phase with the dominant mode in the normal-sized output waveguide. This would produce a voltage coupling that is approximately N times larger than that of a similar post in empty waveguide, where N is the number of unshorted small-height waveguides. However, at frequencies removed from that at which the individual waveguide height d is equal to an odd number of half-wavelengths, this phase relationship is lost and the voltage coupling will be less. An analysis of an idealized system that considers only the effects produced by these phase changes shows that the voltage coupling is given by

$$C = C_0 \tan \frac{\pi d}{\lambda} \sin \frac{N\pi d}{\lambda} \quad \text{if } N \text{ is even} \quad (1)$$

$$C = C_0 \tan \frac{\pi d}{\lambda} \cos \frac{N\pi d}{\lambda} \quad \text{if } N \text{ is odd} \quad (2)$$

where C_0 is the coupling between the post and one small-height waveguide, and λ is the wavelength of the slow wave in the semiconductor. These equations indicate that the coupling can be greatly increased by use of many small-height waveguides.

The system of small-height waveguides illustrated in Fig. 1(a) was fabricated in the following way. Alternate layers of copper-adhesive-clad Mylar and bare Mylar (obtained from the G. T. Schjeldahl Company, Northfield, Minn.) were bonded together in a standard heated platen bench press as illustrated in Fig. 2. The bonded stack was machined to the same cross-sectional dimensions as X-band (8.2–12.4 GHz) waveguide and a length of 1.67 inches. It was then copper-plated over its entire external surface. The copper at the output end of the coupler was milled off, alternate layers were short-circuited, and a waveguide flange was attached to accommodate the empty output waveguide. The periodicity of this laminate was exceptionally good. The average spacing near one end was 4.64 mils, while near the other end, 95 layers away, it was 4.59 mils. In other words, the systematic error in the spacing over 95 layers was approximately 0.05 mil. This relatively small systematic error indicates that one can construct laminates that contain over 100 layers without losing significant phase coherence. A calculation involving the random errors indicates that they will not degrade the voltage coupling by more than 1 percent. Microwave measurements at 10 GHz indicate that a wave can propagate in the small-height waveguides for a distance of about 1.5 inches before losing half of its power. When the structure was cooled to 77°K (liquid nitrogen), the half-power distance was about 4 inches. Since copper has a considerably higher electrical conductivity at 77°K, while the electrical properties of Mylar are virtually temperature-independent, it is obvious that the major power loss was in the copper waveguide walls—exactly as expected for a small-height waveguide with a large surface-to-volume ratio.

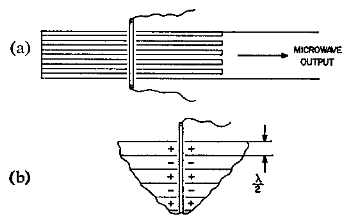

Fig. 1. (a) Schematic illustration of the slow-wave coupler. (b) Phase variation of coupling in alternate small-height waveguides.

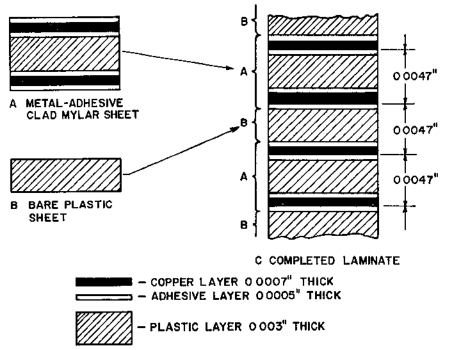

Fig. 2. Schematic illustration of the technique of laminating the actual slow-wave coupler shown in Fig. 1(a).

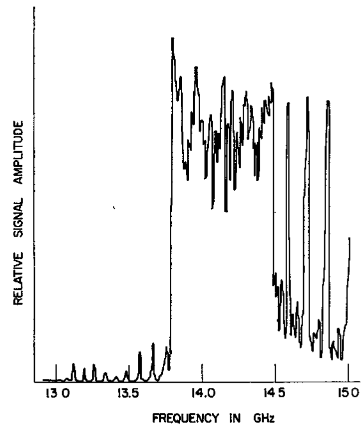

Fig. 3. Spectrum of the microwave emission from an indium antimonide post mounted in the slow-wave coupler of Fig. 1(a).

A 15-mil-diameter hole was drilled in the center of this structure, and a cylindrically shaped bar of high-purity single-crystal N-type indium antimonide of 11-mil diameter was mounted in the hole and insulated from the copper layers by 0.5-mil Mylar sheet. Indium contacts were soldered to the ends of the bar which protruded from this hole. The system was cooled to 77°K and immersed in a magnetic field of 6 kG. The crystal was subjected to an applied voltage pulse sufficient to cause impact ionization breakdown. Under these conditions indium antimonide contains a large plasma density of electrons and holes [1] and emits a noisy mode of microwave emission [2]–[11]. It was hoped that by using this coupler, one could determine whether this microwave emission was due to slow waves in the indium antimonide which have a phase velocity about equal to the average electron drift velocity [12]–[14]. It was observed that the spectrum from 8.5 to 15 GHz was dominated by white noise. However, as shown in Fig. 3, the noise emission between 13.0 and 13.5 and between 14.5 and 15.0 GHz seemed to be absent and in its place a system of periodic peaks was observed. This is exactly the spectral dependence predicted by the above equations. An analysis of these peaks indicates that the main peak should be at about 15.8 GHz (out of range of the microwave receiver used) and would correspond to a mode of operation with five half-wavelengths per laminate spacing. Assuming the existence of a slow-wave mechanism of microwave emission, the spacing between peaks indicates a phase velocity of 7.4×10^7 cm/s compared to an estimated average electron drift velocity of 4.5×10^7 cm/s in the applied 188 V/cm electric field [15]. However, the unenhanced power level of the microwave emission from these waves in the absence of the coupler must be very small compared with the power level of the noisy component of microwave emission, which presumably results from some mechanism other than slow waves [16]. It is also significant that no periodic peaks were observed in the spectrum of the microwave emission from indium antimonide operating below the threshold of impact ionization [17].

A method of coupling slow waves to a waveguide has been found and used to detect very-low-power slow waves in indium antimonide subjected to the combined effects of comparatively high applied electric and magnetic fields.

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References

[1] R. Bowers, "Plasmas in solids," *Sci. Am.*, vol. 209, pp. 46–53, November 1963.
[2] R. D. Larrabee and W. A. Hicinbothem, Jr., "Observation of microwave emission from indium antimonide," *Proc. of Symp. on Plasma Effects in Solids*. Paris: Dunod, 1965, pp. 181–187. (Symposium held in connection with 7th Internat'l Conf. on Physics of Semiconductors, Paris, France, 1964.)
[3] K. Suzuki, "The generation of microwave radiation from InSb," *Japan. J. Appl. Phys.*, vol. 4, pp. 42–52, January 1965.
[4] T. Suzuki, "Microwave emission and low frequency instabilities in InSb," *Japan. J. Appl. Phys.*, vol. 4, pp. 700–701, September 1965.
[5] B. Ancker-Johnson and M. F. Berg, "Microwave emission from *P*-type indium antimonide," *Bull. Am. Phys. Soc.*, ser. II, vol. 10, p. 720, September 1965.
[6] M. Toda, "Saturation of the microwave impedance and the occurrence of microwave emission in InSb," *Japan. J. Appl. Phys.*, vol. 4, pp. 854–860, November 1965.
[7] B. Ancker-Johnson and M. F. Berg, "Microwave radiation from electron-hole plasmas," presented at 7th Ann. Meeting of Division of Plasma Physics, American Physical Society, San Francisco, Calif., November 1965; page S7 of *Program and Abstracts* of papers.
[8] R. D. Larrabee and W. A. Hicinbothem, Jr., "Current oscillations and microwave emission in indium antimonide," *IEEE Trans. on Electron Devices*, vol. ED-13, pp. 121–131, January 1966.
[9] K. Suzuki, "The spectrum of microwave radiation from InSb," *IEEE Trans. on Electron Devices*, vol. ED-13, pp. 132–136, January 1966.
[10] H. Morisaki and Y. Inuishi, "Harmonic generation in microwave emission from InSb," *Japan. J. Appl. Phys.*, vol. 5, pp. 343–344, April 1966.
[11] ——, "Hall mobility and microwave emission in InSb plasma," *Japan. J. Appl. Phys.*, vol. 5, p. 637, July 1966.
[12] R. Bowers and M. C. Steele, "Plasma effects in solids," *Proc. IEEE*, vol. 52, pp. 1105–1112, October 1964.
[13] M. C. Steele, "Possible explanation for microwave emission from InSb in magnetic fields," *Proc. of Symp. on Plasma Effects in Solids*. Paris: Dunod, 1965, pp. 189–191. (Symposium held in connection with 7th Internat'l Conf. on Physics of Semiconductors, Paris, France, 1964.)
[14] B. Vural and M. C. Steele "Possible two-stream instabilities of drifted electron-hole plasmas in longitudinal magnetic fields," *Phys. Rev.*, vol. 139, pp. A300–A304, July 1965.
[15] M. Glicksman and W. A. Hicinbothem, Jr., "Hot electrons in indium antimonide," *Phys. Rev.*, vol. 129, pp. 1572–1577, February 1963.
[16] M. C. Steele, "Microwave generation from photoconductive mixing of amplified spontaneous radiation," *RCA Rev.*, vol. 27, pp. 263–271, June 1966.
[17] S. J. Buchsbaum, A. G. Chynoweth, and W. L. Feldman, "Microwave emission from indium antimonide," *Appl. Phys. Lett.*, vol. 6, pp. 67–69, February 1965.

Perturbations of the Critical Parameters of Quarter-Wave Directional Couplers

The theory of coupled transmission lines was discussed by Jones and Bolljahn[1] on the assumption that the phase velocities for even and odd modes v_e and v_o were equal to each other. However, when the coupled lines are constructed in the microstrip geometry or on a hard substrate suspended between two ground planes, the phase velocities are in general unequal. As a result, the even- and odd-mode characteristic impedances Z_{0e} and Z_{0o}, as well as the phase velocities, must be adjusted to prescribed values. However, little is known how critical these adjustments are in order to get desired coupler performance.

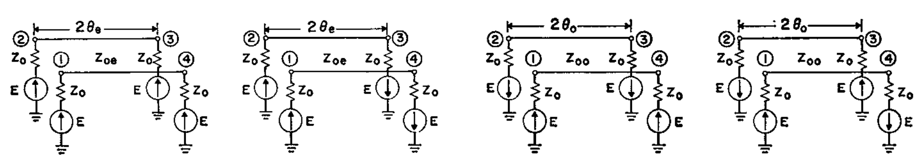
This correspondence reports some results obtained for the scattering coefficients S_{ij} of 3-, 10-, and 20-dB quarter-wave couplers.

For an ideal coupler, $S_{11}=S_{31}=0$ independent of frequency, and S_{21} and S_{41} have their maximum and minimum values at the center frequency ω_0, respectively, where the subscripts refer to port numbers as shown in Fig. 1.

To set up the equations for the scattering coefficients we shall make use of the double symmetry of the two parallel lines. Energy from a generator with a voltage 4E and a source impedance Z_0 will be fed into port 1 in four steps and then superimposed. This is shown in Fig. 1. The first two steps represent even-mode excitation, the last two steps odd-mode excitation. The electrical lengths of the coupling section with physical length l are $2\theta_e=\omega l/v_e$ and $2\theta_o=\omega l/v_o$, respectively.

Because of the polarity of the generator voltages, the current is zero at the center of the lines for the configuration in Figs. 1(a) and 1(c), and the voltage is zero for Figs. 1(b) and 1(d). Therefore, at the center, the lines may be open or short-circuited, respectively, without changing the current at each port. The currents into port 1 for the four cases are therefore given by

$$i_1 = \frac{E}{-jZ_{0e}\cot\theta_e + Z_0}$$
$$i_2 = \frac{E}{jZ_{0e}\tan\theta_e + Z_0}$$
$$i_3 = \frac{E}{-jZ_{0o}\cot\theta_0 + Z_0}$$
$$i_4 = \frac{E}{jZ_{0o}\tan\theta_0 + Z_0}. \qquad (1)$$



(a) (b) (c) (d)

(a) and (b) Two states of even-mode excitation of the coupler.

(c) and (d) Two states of odd-mode excitation of the coupler.

Fig. 1.

TABLE I

		10% Change in v_e or v_o	10% Change in Z_{0e} or Z_{0o}
3-dB Coupler	Return loss (VSWR)	28 dB (1.08)	32 dB (1.05)
	Directivity	25 dB	29 dB
10-dB Coupler	Return loss (VSWR)	33 dB (1.04)	27 dB (1.09)
	Directivity	13 dB	26 dB
20-dB Coupler	Return loss (VSWR)	42 dB (1.02)	26 dB (1.1)
	Directivity	2 dB	26 dB

TABLE II

	Type of Coupler	Permissible v_e or v_o	Change in Z_{0e} or Z_{0o}
For 1.05 input VSWR	3 dB	6.5%	10.0%
	10 dB	11.0%	5.5%
	20 dB	+45% / −25% *	5.5%
For 30-dB directivity	3 dB	6.0%	9.0%
	10 dB	1.5%	6.5%
	20 dB	0.4%	6.5%

* These values were calculated from the exact equation for S_{11}.

Manuscript received February 6, 1967.

[1] E. M. T. Jones and J. T. Bolljahn, "Coupled-strip-transmission-line filters and directional couplers," *IRE Trans. on Microwave Theory and Techniques*, vol. MTT-4, pp. 75–81, April 1956.